Ontario tax studies·9a

the extrapolation of taxfilers and income

supplementary material
Taxation and Fiscal Policy Branch

Ministry of Treasury
Economics and
Intergovernmental
Affairs

Ontario

Ont. Taxation and Fiscal Policy Branch.

Ontario Tax Studies.9

THE EXTRAPOLATION OF TAXFILERS AND INCOME

ONTARIO TAX STUDIES 9

Supplementary Material

THE EXTRAPOLATION OF
TAXFILERS AND INCOME

Nancy Bardecki

April 1974

Preface

This study documents the methods employed to extra-
polate into the future the number of taxfilers and the level
and composition of income which together form the basis of a
projection of the economic environment to 1980 in Ontario.
The estimates assume the Ontario economy develops at close to
its potential capacity between 1972 and 1980. Various rates
of inflation are employed, ranging from 5 per cent through 8
per cent.

Tests were made within this economic environment of
the impact of the structural changes in the personal income
tax due to indexing to compensate for the effects of inflation.
The results of this tax impact analysis may be found in Staff
Paper, The Dynamic Impact of Indexing the Personal Income Tax,
Ontario Tax Studies 9 for which this document provides supple-
mentary material. As further refinements are developed and more
up to date data become available, the methods and estimates used
in the extrapolation procedure outlined in the following pages
will be revised.

This study, by Nancy Bardecki, updates and improves
upon the methods developed by David Holland and Ena Garmaise
for Tax Reform and Revenue Growth to 1980, Ontario Tax Studies
4, published in 1971.

Contributions to the process of updating and improving
the methods outlined here are welcomed.

B. Jones   B.A.R. Hull
Director
Taxation and Fiscal Policy Branch

April, 1974

Table of Contents

List of Tables

INTRODUCTION

The revenue and incidence effects of the income tax
system are sensitive to both the changing characteristics of
the taxfiling population and changes in the relative importance
of income sources through time. To test the performance of
alternative tax structures within the future full employment
performance of the Ontario economy both the taxfiling popula-
tion and the income sources under different rates of inflation
must be simulated.

The composition of the taxfiling population and the
relative importance of income sources in future years are de-
rived using simple trend analysis. Information concerning
groups of taxfilers with various demographic characteristics
is derived through simple functional relationships and demo-
graphic patterns estimated through time. This technique relies
on the high correlation of these variables over time. [1]

[1] For a more complete discussion of the background to the
methodology see Ontario Tax Studies 4: <u>Tax Reform and
Revenue Growth to 1980</u> (Toronto: Ministry of Treasury,
Economics and Intergovernmental Affairs, 1971) Appendix
B page 85.

II. THE THEORY OF THE MODEL FOR TREND ANALYSIS OF THE DISTRIBUTION OF TAXFILERS

In its abstract form, the extrapolation model uses a breakdown of the taxfiling sample according to age, sex and occupational characteristics into cell collections of individuals exhibiting similar characteristics. In the absence of the constraint of data limitations and the need for efficient estimates, time series regressions could be run using the taxfilers in each individual cell as the dependent variable and the total number of taxfilers as the explanatory variable. Extrapolations using the estimated growth parameters could then be made.

This fully disaggregated system may be represented symbolically: let the number of people in a cell be represented by X_{ijk}, where i refers to the age classification, j the sex classification, and k the occupation classification with n_1, n_2, and n_3 units respectively.

Then $\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \sum_{k=1}^{n_3} X_{ijk}$ represents the entire population. For purposes of notational simplicity, let $X = X_{ijk}$ be a three dimensional array containing the $(n_1 \cdot n_2 \cdot n_3)$ cells which represent the population sample.

The projected cell population, as a function of the number of tax returns can be represented as:

$$X_{ijk} = f_{ijk} (XNTR_t)$$

where the function f_{ijk} is determined by a technique described later in this paper.

When projecting from t=0 as a base year, a ratio

$$Y_{ijk}^{t} = f_{ijk} \frac{(XNTR),}{X_{ijk}^{o}}$$

can be used as a multiple on X_{ijk}^{o} to extrapolate to year t. If $Y^{t}=(Y_{ijk}^{t})$

is set up as a three dimensional array, similar in form to X, then

$X^{t} * X^{o}$, where the operator, *, is defined as $(X_{ijk})*(Y_{ijk}) = (X_{ijk} \cdot Y_{ijk})$

for each ijk, i.e., simple multiplication of corresponding cells.

The abstract form of this model is modified for estimating

purposes because of data limitations and to take advantage of economies

of aggregation. Since the data may be aggregated without significant

information loss, the estimation is conducted in a more concise form.

For the concise approach to be valid, independence among the

three factors for a given individual must be assumed. The independence

assumption means that the function f_{ijk} may be represented as a simple

product of its three marginal components, such that:

$$f_{ijk} = f_{i} \cdot f_{j} \cdot f_{k}.$$

A Y-array generated in this manner requires only $n_1 + n_2 + n_3$ regressions.

Under the condition of complete independence, the above equation holds

exactly and there is no information loss. Where the proportion of tax-

filers within a cell with a given characteristic is large relative to

the numbers with that characteristic in the taxfiling population as a

whole, bias will exist. For example, if a large number of filers who

are over 65 are also pensioners, both the f (pensioners) and the f (65

and over), will represent the rate of growth of a largely similar group
and over-estimation will occur in this cell. The resultant error would
vary directly as the square of the difference between the actual f_{ijk}
parameters and the parameters estimated under the above formula.[2]

Under the assumption of independence the model is:

$$X^t_{ijk} = Y^t * X^o$$

where Y^t represents $\dfrac{f_i f_j f_k \quad (\text{XNTR})}{X^o}$

---

[2]

The concept is best illustrated by using the statistical technique
of "analysis of variance" where the object is to explain those
factors which cause deviations from the mean. The procedure used
was a factorial experiment design which is represented, in two
dimensions, by the model.

$$Y_{ijk} = U + \alpha_i + \beta_j + \lambda_{ij} + \epsilon_{tij},$$

$$\Sigma \alpha_i = \Sigma \beta_j = \Sigma \lambda_{ij} = 0$$

where α_1 represents the influence of the rows, βj the columns, and λ_{ij}

the interaction of the two. For the model, u represents the general
population inflation, α_i and β_j and λ_{ij}, the influence of the various
factors. The basic data undergoe two transformations: one, a logarithmic
transformation to make the effects additive, since the model is multi-
plicative, and two, and appropriate orthonormalization to put the data
in a form for an analysis of variance. This being done, the sums of
squares may be examined to discover the sources of variation.

The variance $\Sigma (Y_{tij} - \overline{Y})^2$
may be broken down into variance due to rows $TJ\Sigma_i (Y._i. - \overline{Y})^2$

variance due to interaction $T\Sigma_i \Sigma_j (Y._{ij} - Y._i. - Y.._j - \overline{Y}_{t..})^2$

and unexplained variance $\Sigma_t \Sigma_i \Sigma_j (Y_{tij} - \overline{Y}_{t..\ jk})^2$

The loss of information due to the independence assumption is therefore
equal to the interaction term. Implicity, it is assumed that the mean
square of the interaction has in insignificant F-test with respect to
the error term, and thus, that the error is ignoring its effect is small.
For a concise discussion of the relationship of this technique to
econometrics see, A. S. Goldberger, Econometric Theory (New York, Wiley,
1964), pages 227-231.

III. THE MATHEMATICS OF GENERATING THE Y ARRAY

A. Estimation of Demographic Growth Parameters for Ontario

The Software

The 1972 edition of the Bank of Canada's Massager Program as offered by Computer Science Corporation is used to run regressions.

The Functional Form of the Equation

A linear function is too rigid for estimating changing demographic patters; it is intuitively unreasonable to expect that demographic variables will increase proportionately with some exogenous variable over time.

Thus, the following functional form is used:

$$Y = \alpha \cdot X^{\beta}; \alpha > 0$$

where

Y = the dependent variable

X = the explanatory variable

α, β = growth parameters[3]

---

[3] This functional form is consistent with the assumption of constant elasticity of response between the dependent and explanatory variables. Examination of the derivatives allows an inductive prediction of the relationship between X and Y when the size and sign of the parameter is known.

1st Derivative:

$$Y' = \alpha \cdot \beta \cdot X^{\beta-1}$$

Y is an increasing function of X where $\beta > 0$
Y is a constant function of X where $\beta = 0$
Y is a decreasing function of X where $\beta < 0$

2nd Derivative:

$$Y'' = \alpha \cdot \beta \cdot (\beta-1) \cdot X^{\beta-2}$$

Y is concave from above where $\beta > 1$
Y is linear and increasing where $\beta = 1$
Y is convex but montonically increasing where $1 > \beta > 0$
Y is 0 as an asymtote where $\beta < 0$

Statiscally, the parameters are estimated using a logarithmic transfor-
mation of the data; the estimation equation becomes:

$$logY = log\ \alpha + \beta\ logX + logU$$

Where reference to the Durbin-Watson statistic indicates that serial
auto-correlation is present, remedial action is taken. If left untreated,
predictions would be inefficient due to unnecessarily large sampling
variances.

When first order serial-autocorrelation appears, the distur-
bance term is as follows:

$$logU_t = \rho logU_{t-1} + log\varepsilon$$

or more generally

$$logU_t = \rho^t logU_{t-n} + \sum_{k=o}^{n} \rho^k log\varepsilon_{t-k}$$

where:

$\rho^t logU_{t-n}$ = serially dependent error

$\sum_{k=o}^{n} \rho^k log\varepsilon_{t-k}$ = serially independent error which
conforms to all assumptions of the
general linear model.

n = number of years from the base period

ρ = parametric relationship between the error in the
present period and the error in the previous period.

The estimation equation is:

$$logY_t = log\alpha + \beta logX_t + \rho logU_{t-1} + log\varepsilon_t$$

or

$$logY_t - \rho logY_{t-1} = log\alpha(1-\rho) + \beta(logX_t - \rho logX_{t-1}) + log\varepsilon_t$$

The Massager Program, allows the simple application of the Hildreth-Lu technique of correcting serial autocorrelation. Operation 67 estimates ρ, transforms the data, then automatically re-estimates the co-efficients, α and β.

The prediction equation takes the following form:

$$\log Y_t = \log \alpha + \beta \log X_t + \rho^t \log U_o$$

To obtain the multiplicative form of this equation, the exponential of both sides is taken to yield:

$$Y_t = \alpha . X^\beta . U_o^{\rho^t}$$

Estimation of the Growth Parameter for the Number of Taxfilers

The dependent variable, the number of tax returns (XNTR), is regressed on the employed labour force (ELF). Observations are for the years 1951 to 1971. Data for XNTR are obtained from Taxation Statistics[4] Table 8, "All Returns by Province and Income" and data for ELF are obtained from the CANSIM tape.

---

[3] $\log U_o = \log Y_o - \log \hat{Y}_o$

$$= \log (Y_o / \hat{Y}_o)$$

Taking the exponential of both sides of this expression, the following is derived:

$$U_o = Y_o / \hat{Y}_o$$

[4] Taxation Statistics, (Ottawa: Information Canada, Department of National Revenue), annual publications 1955-1973.

Estimation of Growth Parameters for Age

Four age groups are considered:

1. under 25 years

2. 25-39 years

3. 40-64 years

4. 65 years and over

The dependent variable, the specified age group, is regressed on XNTR. Observations are for the years 1951-1971. The number of taxfilers in each age group is obtained from Taxation Statistics, Table 10A, "All Returns by Age and Province".

Estimation of the Growth Parameters for Sex

The taxfiling population is divided according to sex. The dependent variable, male or female filers, is regressed on XNTR. Observations are for the years 1951-1971. The number of taxfilers in each sex classification for Canada is obtained from Taxation Statistics, Table 4, "All Returns by Sex and Age".[5]

Estimation of Growth Parameters for Occupation

Eight occupational groups are used:

1. farmers and fishermen
2. professionals
3. business proprietors
4. salesmen
5. investors
6. pensioners
7. employees; and
8. unclassified[6]

---

[5] Ontario data are not available; therefore it is assumed that the rates of growth of male and female taxfilers in Ontario is similar to the rates of growth in Ontario.

[6] In 1972 and thereafter, unclassified includes the military. The military and other unclassified returns are assumed to grow at the general growth rate for taxfilers.

Table 3:1 describes the occupations included in these broad categories.

The dependent variable, the specified occupational group, is regressed on XNTR. Observations are for the years 1963-1971. The number of taxfilers in each occupational group is obtained from Taxation Statistics, Table 3, "All Returns by Occupation".[7]

The results of the above regressions are shown in Tables 3:2 and 3:3.

---

[7]
  Ontario data are not available. It is assumed that the rate of growth for the given occupational groups in Ontario is similar to the rate of growth in Canada.

Table 3:1

Occupation Definitions

Occupational Category	Occupations Included
Farmers & Fishermen	Farmers & Fishermen
Professionals	Accountants; Doctors; Lawyers & Notaries; Engineers; Architects; Entertainers, Artists; Nurses & Other
Business Proprietors	Foresters; Wholesale Trade, Retail Trade; Insurance Agents; Real Estate Agents; Manufacturing; Construction; Public Utilities; Recreational Services; Business Services, Other Services
Salesmen	Salesmen who report commission income from self-employment.
Investors	Investors and Property Owners
Pensioners	Persons whose main source of income is government or company pension.
Employees	Employees of Business Institutions; Federal, Provincial and Municipal Employees; Teachers & Professors; Unclassified Employees
Unclassified	Military; Estates; Those not elsewhere defined.

Source: Taxation Statistics 1972, (Ottawa: Information Canada, 1972) pp. 166-168.

Note: The occupational categories and the definitions of the occupations included are those used by the Department of National Revenue.

Table 3:2

Demographic Growth Parameters
For Age and Sex

Variable Name	logα (t-score)	β (t-score)	ρ	$\bar{R}^2$
All Filers	− 7.515695 (−16.878)	1.516700 (49.950)	.431	.999
Age				
Under 25	−23.709239 (−3.699)	2.473134 (5.772)	.554	.995
25-39	− 4.719152 (−3.522)	1.234667 (13.784)	.202	.998
40-64	− 6.539460 (−2.314)	1.370284 (7.254)	.419	.998
65 and over	−28.208447 (−7.019)	2.712931 (10.098)	.235	.980
Sex				
Male	1.343634 (.847)	.872285 (8.227)	.792	.999
Female	− 3.236589 (−1.577)	1.116029 (8.453)	.803	.999

Source: Regression analysis of data on tax returns in Ontario pro-
vided by Department of National Revenue in their document,
Taxation Statistics (Ottawa: Information Canada, 1953-1973).

Table 8, "All Returns by Province and Income".
Table 10A,"All Returns by Age and Province".
Table 4, "All Returns by Sex and Age".

Table 3:3

Demographic Growth Parameters
For Occupation

Variable Name	logα (t-score)	β (t-score)	ρ	$\bar{R}^2$
Farmers & Fishermen	1.550707 .424	.690801 (3.001)	.666	.999
Professionals	- 1.620540 (-.760)	.794352 (5.918)	.454	.999
Business Proprietors	8.296395 (26.673)	.280859 (14.348)	-.102	.999
Salesmen	- 5.729024 (-1.616)	.992711 (4.472)	.219	.998
Investors	- 17.468805 (-4.533)	1.894792 (7.808)	.708	.999
Pensioners	- 41.283500 (-11.543)	3.381064 (15.012)	.798	.991
Employees	.746570 (1.830)	.940978 (36.633)	.830	.999
Unclassified	- 30.839770 (-3.781)	2.629890 (5.123)	-.108	.778

Source: Regression analysis of data on tax returns in Ontario provided by Department of National Revenus in their document, Taxation Statistics, (Ottawa: Information Canada, 1965-1973).

Table 3, "All Returns by Occupation and Income".

B. Extrapolation of the Number and Distribution of Taxfilers

The extrapolation factors are calculated in a computer pro-
gram developed by Taxation and Fiscal Policy Branch.[8]

Projections of the total number of taxfilers and the number
of taxfilers in the various age, sex and occupational groups are made
for the years 1972-1985 using the following prediction equation:

$$Y_t = \alpha . \ X_t^{\beta} \cdot U_o^{\rho^t}$$

where:

$$X = \text{explanatory variable}$$
$$\alpha, \ \beta, \ \rho = \text{estimated growth parameters}$$
$$U_o = Y_{1971} \div \hat{Y}_{1971}$$

Table 3:4 shows the parameters α, β, and ρ and the estimated
Y_o values which the program uses to calculate U_o.

Prediction of Number of Taxfilers

For the prediction of the total number of tax returns, an
estimate of the explanatory variable, ELF is required. The ELF is as-
sumed to·grow at the rates indicated by Wolfgang Illing in his study
for the Economic Council of Canada.[9] An unemployment rate of 4 per
cent is assumed to persist throughout the projection interval.

A normalization adjustment, which is fully described in the
following section, is made to compensate for the structural change
which occurred in 1972. Then the number of taxfilers in the years
1973 to 1985 is expressed as a fraction of the number of taxfilers in
1972 in order to obtain the growth factor.[10]

[8] David Holland, XNTR, "The taxfiler and Income Sources Extrapola-
tion Simulator" (August, 1970).

[9] Illing, W., "Population, Family, Household and Labour Growth to
1980", study prepared for the Economic Council of Canada (Ottawa:
Information Canada, 1967).

[10] In this case and hereafter, growth refers to the cumulative growth
which occurs from 1972.

Prediction of Demographic Characteristics of Taxfilers

The explanatory variable which is used for the prediction of the number of taxfilers in the various age, sex, and occupational classes, is generated within the program.

The sum of the groups within each of the broad categories; age, sex and occupation, constitutes a closed set of taxfilers and the groups within each broad demographic category are mutually exclusive. These groups are normalized ensure that the sum of taxfilers in each broad demographic category equals the total number of taxfilers.

$$\text{Normalization Factor} = \frac{XNTR}{\sum\limits_{i=1}^{n} Xi}$$

where

Xi are the estimated values for either the age, sex, or occupational groups.

A second normalization adjustment is made to compensate for the structural change which occurred in 1972.[11]

Actual data on the number of taxfilers and their distribution across the various age, sex and occupational groups in 1972 are obtained from the preliminary Green Book sample of 1972 tax returns for Ontario. Actual values for the age, sex and occupational groups are divided by the corresponding estimated values for 1972 to yield adjustment factors. These factors are applied to the predictions in ensuing years.

---

[11] Reforms such as the introduction of the Ontario Tax Credit System caused an increase in the ratio of taxfilers to total population, the increase in taxfilers was proportionally greater for some categories than for others. Persons who did not file tax returns previously, mainly pensioners, students, etc. had to file to claim entitlement to the Property Tax Credit. In addition, some tax-filers are added to the rolls due to the taxation of Unemployment Insurance Benefits.

The number of taxfilers in the various age, sex and occupa-
tional groups in the years 1973-1985 are expressed as a fraction of
the number of taxfilers in these same groups in 1972 in order to obtain
growth factors.

These factors are the f_i's, f_j's, and f_k's discussed in
Section II. Since there are four age groups, two sex groups, and
eight occupational groups, there is a possibility of 64 cells or Y_{ijk}.
Each Y_{ijk} is determined by $f_i \cdot f_j \cdot f_k$. However each of these f's contains
information on the trend rate of growth of the taxfiling population as
a whole, $f(gen)$. To avoid double counting, each $f_i \cdot f_j \cdot f_k$ is divided
by $(f gen)^2$. [12]

Some cells contain relatively few taxfilers and some degree
of aggregation is feasible. Distinction between classes within the
three major demographic groups may be dropped and thus the 64 possible
$(f_i \cdot f_j \cdot f_k)$'s are represented by 34 cells. Where aggregation proves
practical, the f value for the collapsed characteristic is replaced by
f gen. For example, business proprietors aged 25-39, are not differ-
entiated according to sex. The calculation which yields the growth
rate for this cell is:

$$f_i \ . \ f \ gen. \ f_k$$

where

$$f_i = 25 - 39$$
$$f_k = \text{business proprietors}$$

Since cells which are aggregated are relatively small, only
a minimal loss of information results.

----

[12]

It is assumed that the independence assumption breaks down for
$f_i \cdot f_j \cdot f_k$ where i = over 65 and k = pensioners. To avoid double
counting, the f_i is replaced with f gen. in the calculation of
f_{ijk}.

Table 3:4

Estimated Demographic Input for XNTR

	log α	β	ρ	$\hat{Y}$
Age				
1	-23.709239	2.473134	.554	894,929
2	- 4.719152	1.234667	.202	1,170,077
3	- 6.539460	1.370284	.419	1,463,849
4	-28.208447	2.712931	.235	381,582
Sex				
1	1.343634	.872285	.792	2,110,425
2	- 3.236589	1.116029	.803	1,644,686
Occ				
1	1.550707	.690801	.666	305,178
2	- 1.630540	.794352	.454	69,376
3	8.296395	.280859	-.102	365,810
4	- 5.729024	.992711	.219	27,694
5	-17.468805	1.894792	.708	452,499
6	-41.283500	3.381064	.798	481,032
7	.746570	.940978	.830	7,744,163
8	-30.839770	2.629890	-.108	91,540
All Filers	- 7.515695	1.516700	.431	3,783,462

Source: Regression analysis of data on tax returns in Ontario pro-
vided by Department of National Revenue in their document,
Taxation Statistics, (Ottawa: Information Canada, 1953-
1973).

Table 10A, "All Returns by Age and Province".
Table 4, "All Returns by Sex and Age".
Table 3, "All Returns by Occupation and Income".
Table 8, "All Returns by Province and Income".

IV. EMPIRICAL RESULTS

Table 4:1 gives the growth rates established for the various demographic groups. Table 4:2 sets out the growth rates for 1974 and 1980 and Table 4:3 shows the predicted number of taxfilers in each of the groups in 1974 and 1980. While growth rates and absolute numbers are interesting in themselves, it is also interesting to see how the differing growth rates affect the relative size of the groups within the various demographic classes. Several trends can be seen by examining Table 4:4. The proportions of taxfilers under 25 and particularly those 65 and over are increasing while the proportion of taxfilers age 25-64 is falling. Females will constitute an increasingly greater proportion of all taxfilers. Trends show that farmers, professionals, and small business proprietors will constitute a decreasing proportion of the taxfiling population while the numbers of investors and pensioners are increasing in relative importance. Other groups are maintaining a relatively constant position.

Table 4:5 defines the cell classifications of taxfilers following aggregation. Table 4:6 lays out the growth rates predicted for these groups for the years 1974-1980. Table 4:7 reveals the distribution of taxfilers in 1972 as well as the prediction of the distribution of taxfilers in 1974 and 1980. These predicted distributions are made on an aggregated basis rather than by the micro simulation model, TISIM; however, the general pattern of distribution will be similar. All taxfilers under 25 constitute a growing proportion of the taxfiling population. While employees still constitute the major proportion of taxfilers, their number in all age groups except those 65 and over is falling. Trends suggest that all pensioners and investors, particularly those in the 65 and over age group, will constitute a growing proportion

of taxfilers.   Other cells of taxfilers make up a relatively small part
of the taxfiling population but a brief examination of trends in inter-
esting.   The relative importance of all farmers in the 25-29 age group
and of male professionals in the age 40-64 age group drops rapidly.
All other groups seem to maintain a relatively constant position in the
distribution of taxfilers.   Table 4:8 facilitates comparison of the
growth rates of some occupational groups according to age and sex.

Table 4:1

Growth Factors of Various Demographic Groups

1974 - 1980

	1974	1975	1976	1977	1978	1979	1980
All taxfilers	1.0732	1.1141	1.1518	1.1915	1.2331	1.2762	1.3209
Occupation							
Farmers	1.0681	1.1024	1.1327	1.1636	1.1948	1.2264	1.2584
Professionals	1.0439	1.0753	1.1051	1.1369	1.1702	1.2046	1.2399
Small Business	1.0254	1.0387	1.0506	1.0627	1.0749	1.0872	1.0995
Salesmen	1.0875	1.1318	1.1722	1.2148	1.2591	1.3051	1.3526
Investors	1.1476	1.2346	1.3172	1.4073	1.5043	1.6082	1.7193
Pensioners	1.2713	1.4446	1.6180	1.8168	2.0424	2.2970	2.5837
Employees	1.0738	1.1150	1.1527	1.1924	1.2337	1.2765	1.3206
Unclassified	1.0732	1.1141	1.1518	1.1915	1.2331	1.2762	1.3209
Age							
Under 26	1.0887	1.1488	1.2079	1.2740	1.3457	1.4223	1.5034
25-39	1.0608	1.0848	1.1039	1.1217	1.1387	1.1550	1.1708
40-64	1.0499	1.0748	1.0967	1.1187	1.1405	1.1621	1.1834
65 and over	1.1445	1.2352	1.3197	1.4099	1.5055	1.6067	1.7137
Sex							
Male	1.0699	1.1078	1.1423	1.1782	1.2151	1.2531	1.2920
Female	1.0770	1.1216	1.1628	1.2071	1.2539	1.3031	1.3546

Source: Extrapolations using the computer Model, XNTR.

Note 1. The growth factor is calculated by expressing the number of taxfilers in a group in a given year as a fraction of the number of taxfilers in the corresponding group in 1972.

Note 2. In 1972 and thereafter, the military is included in the unclassified group; therefore the general growth rate is used.

Table 4:2

The Distribution of Taxfilers Across the
Various Age, Sex, and Occupational Classes

	Actual 1972 Numbers	Projected Growth Rates	
		1974/1972 (%)	1980/1972 (%)
Age			
Under 25	1,005,265	8.87	50.34
25-39	1,132,818	6.08	17.08
40-64	1,490,282	4.99	18.34
65 and over	499,606	14.45	71.37
Sex			
Male	2,471,590	6.99	29.20
Female	1,656,380	7.70	35.46
Occupation			
Farmers	70,040	6.81	25.84
Professionals	27,680	4.39	23.99
Business Proprietors	127,350	2.54	9.35
Salesmen	8,020	8.75	35.26
Investors	214,720	14.76	71.93
Pensioners	332,650	27.13	158.37
Employees	3,158,230	7.38	32.06
Unclassified	189,280	7.32	32.09
Total	4,127,970	7.32	32.09

Source: The distribution of taxfilers in 1972 is obtained from an
Ontario Treasury computer analysis of a preliminary Green
Book Sample of Ontario Tax Returns. Growth rates are pre-
dicted by the computer program XNTR.

Note: In 1972 and in following years, the military is included in
the unclassified group; therefore, the general growth rate
is used.

Table 4:3

Projected Taxfiling Population
According to Demographic Characteristics

	Actual 1972	Projected 1974	Projected 1980
Age			
Under 25	1,005,265	1,094,432	1,511,315
25-39	1,132,818	1,201,693	1,326,303
40-64	1,490,282	1,564,647	1,763,600
65 and over	499,606	571,799	856,175
Sex			
Male	2,471,590	2,644,354	3,193,294
Female	1,656,380	1,783,921	2,243,732
Occupation			
Farmers	70,040	74,810	88,138
Professionals	27,680	28,787	34,320
Small Business Proprietors	127,350	127,351	140,186
Salesmen	8,020	8,722	108,479
Investors	214,720	246,413	369,168
Pensioners	332,650	422,898	859,468
Employees	3,158,230	3,391,307	4,170,759
Unclassified	189,280	203,135	399,892
TOTAL	4,127,970	4,430,075	5,452,720

Source: The distribution of taxfilers in 1972 is obtained from a com-
puter analysis of a preliminary Green Book Sample of Ontario
taxfilers. Projected distributions are calculated using rates
established by the computer program XNTR.

Note 1. In 1972 and in following years, the military is included in
the unclassified group; therefore, the general growth rate
is used.

2. These predicted distributions are calculated using an aggrega-
tive model; they are not the distributions which are derived
by the microsimulation model.

Table 4:4

Projected Distribution of
the Taxfiling Population

	Actual 1972 (%)	Projected 1974 (%)	Projected 1980 (%)
Age			
Under 25	24.35	24.70	27.72
25-39	27.44	27.12	24.32
40-64	36.10	35.32	32.34
65 and over	12.10	12.91	15.70
Total	100.00	100.00	100.00
Sex			
Male	59.87	59.69	58.73
Female	40.13	40.27	41.27
Total	100.00	100.00	100.00
Occupation			
Farmers	1.70	1.69	1.62
Professionals	.67	.65	.63
Small Business Proprietors	3.08	2.87	2.57
Salesmen	.19	.20	.20
Investors	5.20	5.56	6.77
Pensioners	8.06	9.55	15.76
Employees	76.51	76.50	76.49
Unclassified	4.59	4.59	4.59
Total	100.00	100.00	100.00

Source: The 1972 distribution of taxfilers is derived from an Ontario
         Treasury computer analysis of the preliminary Green Book Sample
         of tax returns for Ontario. The projected distributions are
         calculated using growth rates generated by XNTR.

Note 1. These proportions are calculated using an aggregative model and
         will differ slightly from those calculated using the micro-
         simulation model.

2. Sums may not add to totals due to rounding.

Table 4:5

Cell Classification of Taxfilers
by Age, Sex and Occupation

Class	Age	Sex	Primary Occupation
1	Under 25	Male	All
2		Female	
3	Age 25-39	Male	Employee
4		Female	Employee
5		Both	Farmer
6		Male	Professional
7		Female	Professional
8		Both	Business Proprietor
9		Both	Salesman
10		Male	Investor
11		Female	Investor
12		Male	Pensioner
13		Female	Pensioner
14	Age 40-64	Male	Employee
15		Female	Employee
16		Both	Farmers
17		Male	Professional
18		Female	Professional
19		Both	Business Proprietor
20		Both	Salesman
21		Male	Investor
22		Female	Investor
23		Male	Pensioner
24		Female	Pensioner
25	Age 65 and over	Male	Employee
26		Female	Employee
27		Both	Farmer
28		Both	Professional
29		Both	Business Proprietor
30		Both	Salesman
31		Both	Investor
32		Male	Pensioner
33		Female	Pensioner
34	General	General	General [1]

Note 1: Cell 34 represents all ages and both sexes of the unclassified group. In 1972 and in following years, the military is included in the group; therefore, the general growth rate is assumed to hold.

Table 4:6

Growth Rates for Various Demographic Cells
1974-1980

Class	1974	1975	1976	1977	1978	1979	1980
1	1.0854	1.1423	1.1979	1.2597	1.3262	1.3966	1.4705
2	1.0926	1.1565	1.2195	1.2907	1.3685	1.4523	1.5417
3	1.0581	1.0794	1.0957	1.1099	1.1227	1.1343	1.1449
4	1.0652	1.0928	1.1154	1.1372	1.1586	1.1796	1.2003
5	1.0558	1.0733	1.0857	1.0954	1.1033	1.1099	1.1154
6	1.0287	1.0410	1.0504	1.0583	1.0649	1.0704	1.0749
7	1.0355	1.0539	1.0693	1.0843	1.0989	1.1131	1.1269
8	1.0135	1.0113	1.0069	1.0004	.9926	.9839	.9745
9	1.0750	1.1020	1.1235	1.1436	1.1627	1.1811	1.1988
10	1.1309	1.1952	1.2521	1.3100	1.3689	1.4291	1.4905
11	1.1384	1.2101	1.2746	1.3422	1.4126	1.4861	1.5626
12	1.2528	1.3985	1.5379	1.6912	1.8586	2.0411	2.2399
13	1.2611	1.4159	1.5656	1.7328	1.9180	2.1226	2.3483
14	1.0473	1.0695	1.0885	1.1069	1.1245	1.1413	1.1572
15	1.0542	1.0828	1.1081	1.1342	1.1605	1.1869	1.2133
16	1.0450	1.0635	1.0786	1.0924	1.1051	1.1167	1.1274
17	1.0182	1.0315	1.0436	1.0554	1.0666	1.0770	1.0865
18	1.0249	1.0443	1.0624	1.0814	1.1007	1.1200	1.1391
19	1.0032	1.0021	1.0004	.9977	.9943	.9900	.9850
20	1.0640	1.0919	1.1162	1.1405	1.1646	1.1884	1.2118
21	1.1193	1.1843	1.2440	1.3064	1.3711	1.4379	1.5066
22	1.1267	1.1990	1.2663	1.3386	1.4149	1.4953	1.5795
23	1.2400	1.3857	1.5280	1.6866	1.8616	2.0537	2.2641
24	1.2482	1.4029	1.5555	1.7281	1.9211	2.1357	2.3737
25	1.1416	1.2290	1.3099	1.3951	1.4844	1.5779	1.6758
26	1.1492	1.2443	1.3335	1.4294	1.5318	1.6409	1.7569
27	1.1391	1.2221	1.2979	1.3768	1.4588	1.5440	1.6326
28	1.1133	1.1921	1.2662	1.3453	1.4287	1.5165	1.6086
29	1.0935	1.1515	1.2038	1.2574	1.3124	1.3688	1.4265
30	1.1598	1.2548	1.3432	1.4374	1.5373	1.6430	1.7548
31	1.2238	1.3687	1.5093	1.6652	1.8367	2.0247	2.2305
32	1.1410	1.2281	1.3089	1.3941	1.4836	1.5776	1.6762
33	1.1485	1.2434	1.3324	1.4284	1.5310	1.6406	1.7574
34	1.0732	1.1141	1.1518	1.1915	1.2331	1.2762	1.3209

Source: Growth rates generated by the computer model, XNTR.

Table 4:7

Changes in Relative Sizes of the Various
Demographic Cells Through Time

| Class | Actual 1972 | | Predicted 1974 | | Predicted 1980 | |
	Number	% of Total Taxfilers %	Number	% of Total Taxfilers %	Number	% of Total Taxfilers %
1	471,080	11.41	511,310	11.51	692,723	12.59
2	365,640	8.86	399,740	8.90	563,707	10.25
3	709,570	17.19	750,796	16.89	812,387	14.77
4	404,730	9.80	431,118	9.70	485,797	8.83
5	12,840	.31	13,556	.31	14,322	.26
6	8,110	.22	8,342	.18	8,717	.16
7	2,130	.05	2,206	.05	2,400	.04
8	37,850	.92	38,361	.86	36,884	.67
9	1,520	.04	1,634	.04	1,822	.03
10	3,280	.08	3,709	.08	4,889	.09
11	8,430	.20	9,597	.21	13,173	.24
12	60	-	75	-	134	-
13	200	-	252	-	470	-
14	738,040	17.88	772,946	17.39	854,060	15.52
15	437,790	10.60	461,518	10.38	531,171	9.66
16	43,860	1.06	45,834	1.03	49,448	.90
17	11,770	2.85	11,984	.27	12,788	.23
18	2,200	.05	2,255	.05	2,506	.04
19	74,340	1.80	74,578	1.68	73,225	1.33
20	5,320	.13	5,660	.13	6,447	.12
21	24,820	.60	27,781	.62	37,394	.68
22	56,020	1.36	63,118	1.42	88,484	1.61
23	14,170	.34	17,571	.39	32,082	.58
24	12,070	.29	15,066	.34	28,651	.52
25	41,200	1.00	47,034	1.06	68,704	1.25
26	9,630	.23	11,067	.25	16,919	.31
27	11,490	.27	13,088	.29	18,756	.34
28	2,150	.05	2,394	.05	3,458	.06
29	9,660	.23	10,563	.24	13,780	.25
30	780	.02	904	.02	1,368	.02
31	111,830	2.71	136,857	3.08	249,437	4.53
32	157,180	3.81	179,342	4.03	263,465	4.79
33	146,970	3.56	168,795	3.80	258,285	4.69
34	191,240	4.63	205,239	4.62	252,609	4.59

Source: The distribution of tax returns for 1972 is based on a computer
analysis of the preliminary Green Book Sample of tax returns for
Ontario. The predicted distributions are calculated by applying
the growth rates generated by XNTR to the 1972 distribution.

Note: The distributions shown here are those calculated using an
aggregative model, not the distributions which are derived from
the micro-simulation model.

Table 4:8

Growth Rate of Taxfilers by
Age, Sex and Occupation
(percentage growth since 1972)

1974

Occupation

Age	Sex	Employees		Investors		Pensioners	
		M	F	M	F	M	F
26-39		5.81	6.52	13.09	13.84	25.28	26.11[1]
40-64		4.73	5.42	11.93	12.67	24.00	24.82
65 and over		14.16	14.92	22.38		14.10	14.85

1980

Occupation

Age	Sex	Employees		Investors		Pensioners	
		M	F	M	F	M	F
26-39		14.49	20.03	49.05	56.26	123.99	134.83[1]
40-64		15.72	21.33	50.66	57.95	126.41	137.37
65 and over		67.58	75.69	123.05		67.62	75.74

Source: Growth rates are predicted by the computer program XNTR.

Note 1: Pensioners aged 26-64 seem to be experiencing an extremely rapid
         growth rate. These particular cells are near empty and the growth
         rate is insignificant.

V. EXTRAPOLATION OF INCOME SOURCES

Introduction

Since each tax return may report income from various sources, the relative growth of these sources, together with the changing demographic structure of the taxfiling population will have a substantial effect on our distribution of income in the projection years.

Estimation of Growth Parameters

Gross Provincial Product is divided into four major sources of income:

1. Wages and Salaries

2. Non-Farm Unincorporated Business

3. Investment

4. Farm

Since the growth of farm income has fluctuated radically from year to year, growth of this source is estimated using a different technique. The other three income components of GPP have exhibited a stable growth pattern, and growth parameters are determined through a regression analysis. Regressions are run using the following functional form:

$$\log Y = \log \alpha + \log X^{\beta} + \rho^{t} \log U_{0} + \log \varepsilon$$

where Y, the dependent variable, is the given source of income and X, the explanatory variable, is GPP less farm income.

Observations for the years 1951 to 1971 are obtained from the CANSIM tape made available by Statistics Canada. Table 5:1 shows the results of the regressions and the strength of the relationships found.

Extrapolation Procedure

Predictions for the years 1973-1980 are made using the following equation:

$$Y = \alpha. \ (\text{GPP} - \text{farm income})^{\beta} \cdot U_{0} \rho^{t}$$

---

[13]

A complete discussion of this functional form is found in section III A.

Table 5:2 shows the input parameters for the prediction equation.

The explanatory variable, GPP less farm income, is calculated in the following manner. GPP in 1972 is estimated by the Ministry of Treasury, Economics and Intergovernmental Affairs. Real growth of GPP is assumed to be 7% in 1973 and is sustained at 5.6% thereafter. Predictions of GPP are made assuming 5%, 6%, 7%, and 8% inflation. Farm income is subtracted from the GPP predictions in the relevant year.

Farm income in 1972 is derived from Taxation Statistics. The real growth of farm income is assumed to be 3.9% in 1973 and is sustained at 1.6% thereafter. Predictions of farm income are made using 5%, 6%, 7% and 8% inflation.[14]

Tables 5:3-5:6 give the growth factors for various sources of income. It can be seen that investment income is growing much more rapidly than GPP and that wages and salaries are growing slightly faster than GPP whereas the small business and farm income components are growing at a rate much lower than the growth rate of GPP. Some of the disproportionate growth of these various income sources can be explained by Table 4 which shows the rate of growth of taxfilers which generally report these types of income. The rapid growth of investment income is consistent with the rapid growth of investors while the slow growth in small business income may be attributable to

---

[14] Real growth rates of GPP are estimates made by the Ministry of Treasury, Economics and Intergovernmental Affairs. In 1972 farm income was derived by applying the rate of growth of the value of agriculture output given by the Department of Finance to the 1971 gross income of farmers given by Taxation Statistics. The real growth rate of 3.9% for 1973 farm income is obtained from the Dept. of Finance estimate of agricultural output. The sustained real growth of farm income is an estimate made by Ministry of treasury, Economics and Intergovernmental Affairs.

the small growth of business proprietors. However, the growth rate

of farmers is slightly lower than the growth rate of the taxfiling

population and the growth rate of farm income is very low relative

to the growth of GPP. This implies that any province which has a

disproportionate number of farmers will experience a slower growth

in GPP than other provinces.

Table 5:1

Income Source

Growth Parameters

	logα (t-score)	β (t-score)	ρ	$\bar{R}^2$
Income Source				
Wage & Salary	-.903059 -(5.177)	1.031386 (57.769)	.775	.999
Small Business	.403051 (.817)	.698790 (13.831)	.758	.995
Investment	-3.107884 (-4.094)	1.032507 (13.321)	.900	.991

Source: Regression analysis of income data for the years 1951-1971 from CANSIM tape.

Note: The growth of farm income is estimated independently as described in the text.

Table 5:2

Estimated Income Source Input for XNTR

	logα	β	ρ	$\hat{Y}$
Income Source				
Wage and Salary	- .903059	1.031384	.775	21,892
Small Business	.403051	.698790	.758	2,363
Investment	-3.107884	1.032507	.900	2,524

Source: Regression analysis of income data for the years 1951-1971
from the CANSIM tape.

Table 5:3

Income Growth Coefficients
Assuming 5% Inflation

Income Source	1974	1975	1976	1977	1978	1979	1980
Wage & Salary	1.2700	1.4115	1.5690	1.7441	1.9387	2.1551	2.3955
Small Business	1.1778	1.2663	1.3614	1.4635	1.5731	1.6907	1.8168
Investment	1.2941	1.4462	1.6135	1.7979	2.0015	2.2265	2.4755
Farm	1.1803	1.2592	1.3433	1.4330	1.5288	1.6309	1.7398
General	1.2634	1.4010	1.5534	1.7224	1.9098	2.1176	2.3480

Source: Predictions made using the Ontario Treasury Taxfiler and Income
Simulation Model, XNTR.

Note 1. It is assumed that real growth of GPP is 7% in 1973 and is sus-
tained at 5.6% thereafter.

2. It is assumed real growth of farm income is 3.9% in 1973 and is
sustained at 1.6% thereafter.

3. Income Growth coefficients are calculated by expressing income
in the relevant year as a fraction of income in 1972.

4. The rate of inflation is assumed to be 6.5% in 1973 and is sus-
tained at 5% thereafter.

Table 5:4

Income Growth Coefficients
Assuming 6% Inflation

Income Source	1974	1975	1976	1977	1978	1979	1980
Wage & Salary	1.2824	1.4393	1.6154	1.8132	2.0351	2.2842	2.5636
Small Business	1.1856	1.2830	1.3884	1.5024	1.6255	1.7584	1.9019
Investment	1.3067	1.4746	1.6613	1.8692	2.1011	2.3600	2.6494
Farm	1.1916	1.2833	1.3820	1.4884	1.6030	1.7263	1.8592
General	1.2756	1.4278	1.5982	1.7890	2.0025	2.2415	2.5091

Source: Predictions made using the Ontario Treasury Taxfiler and Income
Simulation Model, XNTR.

Note 1. It is assumed that real growth in GPP is 7% in 1973 and is sus-
tained at 5.6% thereafter.

2. It is assumed that real growth in farm income is 3.9% in 1973
and is sustained at 1.6% thereafter.

3. Income Growth coefficients are calculated by expressing income
in the relevant year as a fraction of income in 1972.

4. The rate of inflation is assumed to be 6.5% in 1973 and is sus-
tained at 6% thereafter.

Table 5:5

Income Growth Coefficients
Assuming 7% Inflation

.

Income Source	1974	1975	1976	1977	1978	1979	1980
Wage & Salary	1.2948	1.4673	1.6620	1.8842	2.1353	2.4196	2.7417
Small Business	1.1933	1.2998	1.4158	1.5419	1.6790	1.8280	1.9900
Investment	1.3194	1.5034	1.7100	1.9426	2.2046	2.5001	2.8336
Farm	1.2028	1.3076	1.4215	1.5454	1.6800	1.8264	1.9855
General	1.2876	1.4548	1.6439	1.8574	2.0988	2.3715	2.6796

Source: Predictions made using the Ontario Treasury Taxfiler and Income
Simulation Model, XNTR.

Note 1. It is assumed that real growth of GPP is 7% in 1973 and is sus-
tained at 5.6% thereafter.

2. It is assumed that real growth of farm income is 3.9% in 1973
and is sustained at 1.6% thereafter.

3. Income Growth coefficients are calculated by expressing income
in the relevant year as a fraction of income in 1972.

4. The rate of inflation is assumed to be 6.5% in 1973 and is sus-
tained at 7% thereafter.

Table 5:6

Income Growth Coefficients
Assuming 8% Inflation

Income Source	1974	1975	1976	1977	1978	1979	1980
Wage & Salary	1.3073	1.4955	1.7110	1.9574	2.2393	2.5616	2.9302
Small Business	1.2010	1.3167	1.4434	1.5821	1.7338	1.8998	2.0812
Investment	1.3321	1.5324	1.7597	2.0181	2.3122	2.6470	3.0287
Farm	1.2141	1.3322	1.4618	1.6040	1.7600	1.9312	2.1191
General	1.2996	1.4822	1.6904	1.9279	2.1987	2.5076	2.8599

Source: Predictions made using the Ontario Treasury Taxfiler and Income
Simulation Model, XNTR.

Note 1. It is assumed that real growth of GPP is 7% in 1973 and is sus-
tained at 5.6% thereafter.

2. It is assumed that real growth of farm income is 3.9% in 1973
and is sustained at 1.6% thereafter.

3. Income Growth coefficients are calculated by expressing income
in the relevant year as a fraction of income in 1972.

4. The rate of inflation is assumed to be in 6.5% in 1973 and is
sustained at 8% thereafter.

Table 5:7

Projected Relative Size of Income
Components with 5% Inflation

Income Source	Projected 1972 Income		Projected 1974 Income		Projected 1980 Income	
	$ (000)	% of Total	$ (000)	% of Total	$ (000)	% of Total
Wage & Salaries	34,555	81.69	43,885	82.11	82,777	83.34
Small Business	3,659	8.65	4,310	8.06	6,648	6.69
Investment	3,789	8.96	4,903	9.17	9,380	9.44
Farm Income	297	.70	375	.70	516	.52
Total	42,300	100.00	53,446	100.00	99,320	100.00

Source: Extrapolations of Income using the Ontario Treasury computer model,
         XNTR.

Note 1. The rate of inflation is assumed to be 6.6% in 1973 and sustained
         at 5% thereafter.

     2. Real growth of GPP is assumed to be 7% in 1973 and sustained at
         5.6% thereafter.

     3. Real growth of Farm income is assumed to be 3.89% in 1973 and
         sustained at 1.6% thereafter.

     4. Sums may not add to totals due to rounding.

Table 5:8

Projected Relative Size of Income
Components with 6% Inflation

Income Source	Projected 1972 Income		Projected 1974 Income		Projected 1980 Income	
	$ (000)	% of Total	$ (000)	% of Total	$ (000)	% of Total
Wage & Salaries	34,555	81.69	44,314	82.13	88,587	83.47
Small Business	3,659	8.65	4,338	8.04	6,959	6.56
Investment	3,789	8.96	4,951	9.18	10,038	9.46
Farm Income	297	.70	354	.66	552	.52
Total	42,300	100.00	53,958	100.00	106,135	100.00

Source: Extrapolations of Income using the Ontario Treasury computer
         model XNTR.

Note 1. The rate of inflation is assumed to be 6.5% in 1973 and sus-
         tained at 6% thereafter.

     2. Real growth of GPP is assumed to be 7% in 1973 and sustained
         at 5.6% thereafter.

     3. Real growth of Farm income is assumed to be 3.89% in 1973 and
         sustained at 1.6% thereafter.

     4. Sums may not add to totals due to rounding.

Table 5:9

Projected Relative Size of Income

Components with 7% Inflation

Income Source	Projected 1972 Income		Projected 1974 Income		Projected 1980 Income	
	$ (000)	% of Total	$ (000)	% of Total	$ (000)	% of Total
Wage & Salaries	34,555	81.69	44,743	82.15	94,739	83.58
Small Business	3,659	8.65	4,366	8.02	7,282	6.42
Investment	3,789	8.96	4,999	9.18	10,736	9.47
Farm Income	297	.70	357	.66	590	.52
Total	42,300	100.00	54,465	100.00	113,347	100.00

Source: Extrapolations of Income using the Ontario Treasury computer model, XNTR.

Note 1. The rate of inflation is assumed to be 6.5% in 1973 and sustained at 7% thereafter.

2. Real growth of GPP is assumed to be 7% in 1973 and sustained at 5.6% thereafter.

3. Real growth of Farm income is assumed to be 3.89% in 1973 and sustained at 1.6% thereafter.

4. Sums may not add to totals due to rounding.

Table 5:10

Projected Relative Size of Income
Components with 8% Inflation

Income Source	Projected 1972 Income		Projected 1974 Income		Projected 1974 Income	
	$ (000)	% of Total	$ (000)	% of Total	$ (000)	% of Total
Wage & Salaries	34,555	81.69	45,172	82.17	161,253	82.83
Small Business	3,659	8.65	4,395	7.99	7,616	6.23
Investment	3,789	8.96	5,047	9.18	11,475	9.39
Farm Income	297	.70	361	.66	629	.51
Total	42,300	100.00	54,973	100.00	122,243	100.00

Source: Extrapolations of Income using the Ontario Treasury computer
model, XNTR.

Note 1. The rate of inflation is assumed to be 6.5% in 1973 and sus-
tained at 8% thereafter.

2. Real growth of GPP is assumed to be 7% in 1973 and sustained
at 5.6% thereafter.

3. Real growth of Farm income is assumed to be 3.89% in 1973 and
sustained at 1.6% thereafter.

4. Sums may not add to totals due to rounding.